Estimation of the Parameters of Lorentz Dispersive Media Using a Time-Domain Inverse Scattering Technique

Theseus G. Papadopoulos and Ioannis T. Rekanos
Physics Division, School of Engineering, Aristotle University of Thessaloniki
Aristotle University Campus, Thessaloniki, GR-54124, Greece
thiseasp@auth.gr, rekanos@auth.gr

Abstract — A time-domain inverse scattering technique for estimating the parameters of Lorentz dispersive scatterers is proposed. The estimation of the optical and the static relative permittivity, the resonant frequency, and the damping factor of the scatterer is based on the minimization of a cost function. The latter describes the discrepancy between measured and estimated values of the electric field obtained around the scatterer domain when it is illuminated by wideband excitations. The Fréchet derivatives of the cost function with respect to the scatterer properties are derived analytically. These derivatives can be utilized by any gradient-based optimization algorithm. Numerical results related to the reconstruction of layered Lorentz media show the efficiency of the proposed method.

I. INTRODUCTION

Time-domain inverse scattering methods [1]-[3] have attracted significant interest over the last years because of their applications in medical imaging, geophysical prospecting, nondestructive testing etc. The goal of these methods is to estimate unknown scatterer properties by inverting field measurements obtained when the scatterer is excited by wideband incident waves. In such cases, if the scatterer properties are frequency-dependent, dispersion phenomena appear. Hence, the development of techniques that estimate the parameters of dispersive media is important [4], [5].

In this paper, a method to reconstruct simultaneously the relative optical and static permittivities, the resonant frequency, and the damping factor of Lorentz dispersive media is proposed. The method minimizes the cost function that describes the discrepancy between measured and estimated electric field values. By means of the calculus of variations, we derive the equations of the adjoint problem, which are satisfied by the Lagrange multipliers. Furthermore, the Fréchet derivatives of the cost function with respect to the medium properties are obtained. These derivatives are used by the Polak-Ribière optimization algorithm to estimate iteratively the scatterer properties.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

The relative complex permittivity of an inhomogeneous scatterer exhibiting Lorentz dispersion varies with the angular frequency ω and is given by

$$\varepsilon_r(r,\omega)=\varepsilon_\infty(r)+\frac{\Delta\varepsilon(r)\omega_0^2(r)}{\omega_0^2(r)+2j\omega\zeta(r)-\omega^2} \tag{1}$$

where ε_∞ is the relative optical permittivity, $\Delta\varepsilon=\varepsilon_s-\varepsilon_\infty$ (ε_s is the static permittivity), ω_0 is the resonant frequency, and ζ is the damping factor. It is assumed that the scatterer occupies the domain D, whereas the rest space is free. If D is illuminated by I incident waves and for each incidence the electric field is measured at K positions around D, then $I\times K$ measurements are obtained, which are denoted as $\vec{E}_{ik}^m$ where $i=1,...,I$ and $k=1,...,K$.

We assume that the original scatterer properties are unknown and our objective is to estimate them by inverting the measurements set $\vec{E}_{ik}^m$. This can be achieved by minimizing the cost functional

$$\begin{aligned}F(\mathbf{p},\vec{E},\vec{H},\vec{J},\vec{e},\vec{h},\vec{q}) &= \frac{1}{2}\sum_{i=1}^{I}\sum_{k=1}^{K}\int_0^T\left\|\vec{E}_{ik}-\vec{E}_{ik}^m\right\|^2 dt\\ &+\sum_{i=1}^{I}\int_0^T\int_V[\vec{h}_i\cdot(\nabla\times\vec{E}_i+\mu\partial_t\vec{H}_i)\\ &+\vec{e}_i\cdot(\nabla\times\vec{H}_i-\varepsilon_0\varepsilon_\infty\partial_t\vec{E}_i-\vec{J}_i-\vec{J}_{si})\\ &+\vec{q}_i\cdot(\omega_0^2\vec{J}_i+2\zeta\partial_t\vec{J}_i+\partial_t^2\vec{J}_i-\varepsilon_0\Delta\varepsilon\omega_0^2\partial_t\vec{E}_i)]dvdt\end{aligned} \tag{2}$$

where $\mathbf{p}=[\varepsilon_\infty,\Delta\varepsilon,\omega_0,\zeta]^T$ is a an estimate of the medium parameters, $\vec{J}_{si}$ is the current density that generates the *i*th incidence, $\vec{J}_i$ is the polarization current density inside the dispersive medium, ($\vec{E}_i$, $\vec{H}_i$) are the estimated field solutions when the $\mathbf{p}$ medium parameters are considered, V is the domain of field computation, and T is the time duration of measurement per incidence. The Maxwell's curl equations and the polarization relation are introduced in (2) as equality constraints by means of the Lagrange vector multipliers $\vec{e}_i$, $\vec{h}_i$, and $\vec{q}_i$.

The minimization of (2) requires that its first-order variation is equal to zero. This stationarity condition implies that the Lagrange multipliers $\vec{e}_i$, $\vec{h}_i$, and $\vec{q}_i$ are governed by the following equations:

$$\nabla\times\vec{e}_i-\mu\partial_t\vec{h}_i=0 \tag{3}$$

$$\nabla\times\vec{h}_i+\varepsilon_0\varepsilon_\infty\partial_t\vec{e}_i-\vec{j}_i+\sum_{k=1}^{K}(\vec{E}_{ik}-\vec{E}_{ik}^m)=0 \tag{4}$$

$$\omega_0^2\vec{j}_i-2\zeta\partial_t\vec{j}_i+\partial_t^2\vec{j}_i+\varepsilon_0\Delta\varepsilon\omega_0^2\partial_t\vec{e}_i=0 \tag{5}$$

$$\vec{e}_i\,|_{t=T}=\vec{h}_i\,|_{t=T}=\vec{j}_i\,|_{t=T}=0 \tag{6}$$

where $\vec{j}_i=-\varepsilon_0\Delta\varepsilon\omega_0^2\partial_t\vec{q}_i$. From (3)-(6), it is clear that $\vec{e}_i$, $\vec{h}_i$, and $\vec{j}_i$ satisfy the Maxwell's curl equations and the polarization relation in reversed time. Finally, from the first-order variation of (2), the Fréchet derivatives of the cost function with respect to the medium properties are derived, i.e.,

$$\frac{\delta F}{\delta \varepsilon_{\infty}} = -\sum_{i=1}^{I} \int_0^T (\vec{e}_i \cdot \partial_t \vec{E}_i) dt \tag{7}$$

$$\frac{\delta F}{\delta \Delta\varepsilon} = -\frac{1}{\Delta\varepsilon} \sum_{i=1}^{I} \int_0^T (\vec{j}_i \cdot \vec{E}_i) dt \tag{8}$$

$$\frac{\delta F}{\delta \omega_0} = 2\sum_{i=1}^{I} \int_0^T [\frac{1}{\omega_0} \vec{e}_i \cdot \vec{J}_i - \frac{(2\zeta \vec{j}_i - \partial_t \vec{j}_i) \cdot \vec{J}_i}{\varepsilon_0 \omega_0^3 \Delta\varepsilon}] dt \tag{9}$$

$$\frac{\delta F}{\delta \zeta} = \frac{2}{\varepsilon_0 \omega_0^2 \Delta\varepsilon} \sum_{i=1}^{I} \int_0^T (\vec{j}_i \cdot \vec{J}_i) dt \,. \tag{10}$$

The above derivatives can be utilized by any gradient-based inversion algorithm to iteratively update the scatterer properties. In this work, the Polak-Ribière conjugate gradient algorithm has been adopted.

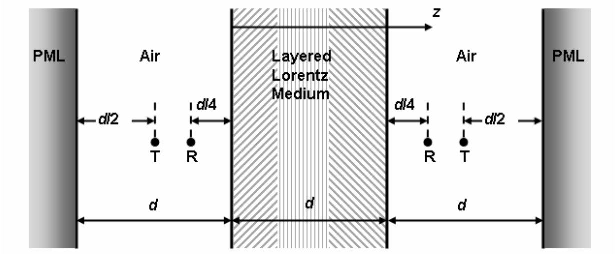

Fig. 1. Geometry of the layered planar Lorentz dispersive scatterer.

III. NUMERICAL RESULTS

The proposed inverse scattering method has been applied to the estimation of the parameters of layered planar Lorentzian media. The geometry of the problem is shown in Fig. 1 where T and R denote the positions of transmitters and receivers, respectively. The total width of the scatterer is $d = 0.15$ m and consists of two layers of equal widths. The properties of the first layer are $\varepsilon_{\infty 1} = 1.5$, $\Delta\varepsilon_1 = 2$, $\omega_{01} = 6\pi \times 10^8$ rad/sec and $\zeta_1 = 0.06\pi \times 10^8$ sec^{-1}, while for the second layer $\varepsilon_{\infty 2} = 2$, $\Delta\varepsilon_2 = 2$, $\omega_{02} = 8\pi \times 10^8$ rad/sec and $\zeta_2 = 0.08\pi \times 10^8$ sec^{-1}. The excitation current density is a modulated Gaussian pulse with central frequency 0.5 GHz and bandwidth 0.5 GHz.

The scatterer reconstruction is based on simulated measurements obtained by use of the FDTD method. The initial estimates of the properties are $\varepsilon_{\infty 1,2} = 1$, $\Delta\varepsilon_{1,2} = 1$, $\omega_{01,2} = \pi \times 10^8$ rad/sec, and $\zeta_{1,2} = 0.01\pi \times 10^8$ sec^{-1}. For all parameters, the estimation errors versus the number of iterations of the Polak-Ribière algorithm are illustrated in Fig. 2. After 1500 iterations all the relative estimation errors are lower than 1.5×10^{-10}, indicating the efficiency of the proposed methodology.

IV. CONCLUSIONS

A time-domain inverse scattering technique for estimating simultaneously all the characteristic parameters of Lorentz dispersive scatterers is proposed. The technique employs the Fréchet derivatives of the cost function, which are derived analytically. Numerical results show that the proposed method gives accurate scatterer reconstruction.

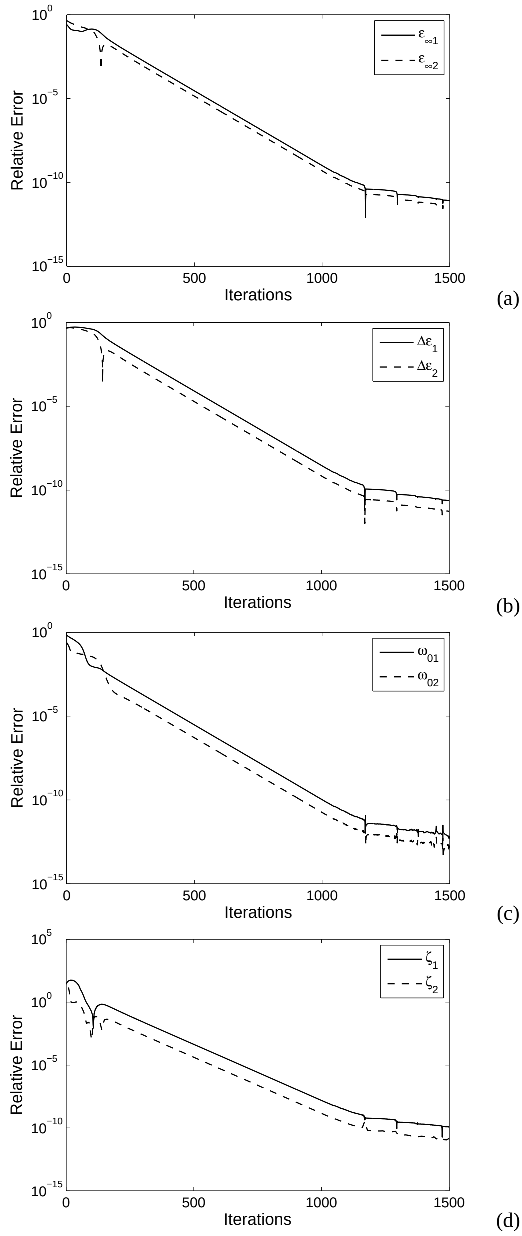

Fig. 2. Relative estimation error of (a) ε_{∞}, (b) $\Delta\varepsilon$, (c) ω_0, and (d) ζ for both layers vs. the number of iterations of the inversion algorithm.

V. REFERENCES

[1] T. Takenaka, H. Jia, and T. Tanaka, "Microwave imaging of electrical property distributions by a forward-backward time-stepping method," *J. Electromagn. Waves Appicat.*, vol.14, no.12, pp. 1609-1626, 2000.

[2] M. Gustafsson and S. He, "An optimization approach to two-dimensional time domain electromagnetic inverse problems," *Radio Sci.*, vol.35, no.2, pp. 525-536, 2000.

[3] I.T. Rekanos and A. Räisänen, "Microwave imaging in the time domain of buried multiple scatterers by using an FDTD-based optimization technique," *IEEE Trans. on Magn.*, vol.39, no.3, pp. 1381-1384, 2003.

[4] S. He, P. Fuks, and G.W. Larson, "An optimization approach to time-domain electromagnetic inverse problem for a stratified dispersive and dissipative slab," *IEEE Trans. on Antennas Propag.*, vol.44, no.9, pp. 1277-1282, 1996.

[5] E. Abenius and B. Strand, "Solving inverse electromagnetic problems using FDTD and gradient-based minimization," *Int. J. Numer. Methods Eng.*, vol.68, no.6, pp. 650-673, 2006.